

NUMBER WORDS AND NUMBER CONCEPTS: THE INTERPLAY OF VERBAL AND NONVERBAL QUANTIFICATION IN EARLY CHILDHOOD

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I. Introduction

A. A KEY QUESTION

The question of how language influences concepts is an old one. Do words merely map onto pre-existing concepts? Or do words actually create the concepts

to which they refer? Would children have notions of different colors, textures, or numbers if they never were exposed to the linguistic labels for these ideas? Interest in these questions can be traced deep into the philosophical roots of psychology and throughout a good deal of the empirical investigation that has been carried out since. However, despite the rhetoric surrounding these issues, we seem to know few actual details of the way language and cognition interact.

This problem is particularly acute in the area of numerical development, for which children must integrate many layers of verbal, procedural, symbolic, and conceptual meaning. To illustrate the complexity involved, consider this 3½-year-old research participant's understanding of "five."

Experimenter: "Can you give me five [blocks]?"

Child: (holds up five fingers) "This is five."

Experimenter: "Can you give me five blocks?"

Child: (lays out 15 blocks and counts them) "One, two, three, four, eight, fifteen. There's five!"

Experimenter: "Okay. Can you count these for me?" (handing the child an array of 10 blocks glued on a board)

Child: "One, two, three, four, eight."

Experimenter: "How many is that?"

Child: "I don't know. Dad, do you know?"

This child has learned what five fingers are and, in that limited context, could be said to understand the concept of "five." Although she confuses "five" and "fifteen" in her count, she seems to know that counting determines cardinal number and that the last word in a count has special meaning. She uses counting to "prove" that her pile of 15 blocks equals "five." But when it comes to performing unfamiliar experimental tasks or counting larger sets, these understandings seem to evaporate. How, then, should we characterize her status? Does she understand "five" or doesn't she? And what does this tell us about the more general process by which children bring meaning to the number words?

In this chapter, we review what is known and what has been proposed regarding the interactions between number words and number concepts. We argue that both classic and current conceptualizations have obscured the rich detail of these interactions by asking, "Which comes first, language or concepts?" As the experimental transcript illustrates, number development viewed close-up is not so orderly. Indeed, we find that this polarizing framework has limited progress in two specific ways: (1) by attempting to separate empirically what cannot be separated developmentally and (2) by casting developmental change in terms of months or years instead of days, or even moments. We conclude the chapter by reviewing case study, microgenetic, and longitudinal research that reveals how fluid and tightly woven the interplay of verbal and nonverbal quantification really is.

B. WHAT DOES IT MEAN TO HAVE A NUMBER CONCEPT?

Like many concepts, number encompasses a variety of perceptual and symbolic inputs. But, it also has aspects that make it unique. For example, consider what it means to understand "five." This notion can be instantiated in groups of objects that vary widely along every other dimension (e.g., five fleas, five skyscrapers, five planets). It can include groups of non-objects, such as sounds, visual events (e.g., lights blinking), actions, ideas, or emotions. These groups may come together in space (e.g., five cookies on a plate), time (e.g., the five cookies I ate last night) or based on function (e.g., the five cookies I know how to bake). The ability to see these diverse groupings as equivalent is a large part of what might be considered nonverbal number concepts.

Number also can be represented symbolically in various ways; as a spoken word (e.g., "five"), a written word, (e.g., five), or as a written numeral (e.g., 5). These symbols for numbers can vary in their intended meaning. Sometimes they simply refer to the number of objects in a set (cardinal meaning). However, they also can refer to a set of measurement units (e.g., five inches, five years, five cups, etc.) or to less standardized measures, such as clothing size (measurement meaning). They are used to denote street addresses, room numbers, radio stations, and so forth, where only position or order matters (ordinality meaning). When they are used in fractions, they can behave differently than they do in reference to whole numbers (e.g., $1/5 < 1/3$ but $5 > 3$). And sometimes these symbols are used as names without quantitative significance, as in license plates and telephone numbers (nominal meaning). (See Fuson (1988, 1992), for an extended discussion of these and other number uses.)

In addition to providing symbols for specific numerosities, verbal numbers also are used in counting. However, counting is conceptually and developmentally distinct from labeling sets. That is, children can count "1-2-3-4-5" without realizing this is the same as determining that a set has five items. Indeed, these ideas remain disconnected for at least a year after children can produce accurate counts (Wynn, 1990, 1992). Furthermore, whereas counting leads to cardinal meaning, the relation between the numbers in a count and the specific objects to which they were applied is arbitrary. Most times, there is no reason that the second item in a count gets the label "two" except that the counter happened to tag that item second. In a subsequent count, the same item could be labeled "four" or "ten" and yet the overall count would yield the same cardinal total as before. Thus, as in other word to referent mappings, the number words are used to tag individual objects; however, in the case of counting, these local pairings are neither stable nor meaningful.

What, then, must children do to develop a concept of number? Certainly, they must come to understand each of the aspects of number outlined here. They must learn to recognize numerical symbols and apply verbal processes, such as

counting, as well as sort out the various meanings and uses that these symbols take on. They must learn to recognize "threeness," "fiveness," and "twenty-fourness" in all possible instantiations. But, perhaps most importantly, they must recognize that all of these components are interrelated. Understanding "five" means knowing that all of these instantiations—five fleas, five planets, five trips, "5," "1-2-3-4-5," and "five"—are the same. To achieve this understanding, children must perform a series of mappings among many situations, skills, and inputs.

Viewed this way, number concepts should develop like other concepts (e.g., dog, blue, shiny), but with added challenges given that (a) the exemplars included in a number category can be vastly different; (b) number categories piggyback on other categories, and (c) there are unique components involved in counting and common number use. For example, to learn the concept of "dog," children have to see that different dogs are all the same because of their "dogness." However, the range of variability for a chihuahua vs. a Great Dane is much narrower than it is for five planets vs. five emotions. Like learning about numerical sets, learning about dogs involves forming an equivalence class and figuring out what to call it. However, for dogs, the equivalence class consists of individual items (i.e., dogs) whereas for number, it consists of sets (i.e., items grouped for some other purpose). Finally, for dogs, there is no analogue to learning the count word sequence. Children need not learn a sequence of animal names and, even if they did, tagging a group of animals with that sequence would not tell them whether or not they had a group of dogs. Thus, number learning likely involves similar processes but greater complexity than learning other concepts.

The present analysis illustrates the many verbal and nonverbal problems children must solve to acquire number concepts—so many, in fact, that it is unlikely they could wholly master one group (verbal or nonverbal) before learning anything about the other. The numerous layers of bootstrapping and meaning on both sides virtually guarantee some amount of bootstrapping. The question, then, is how much bootstrapping and at what level of detail? For individual components? Specific set sizes? Specific contexts? And at what point in development do number words and concepts begin to co-mingle? We first look to two classic positions, as well as the relevant literature specific to number development, to address these questions.

II. The Relation Between Language and Concepts

A. CLASSIC POSITIONS: A FALSE DICHOTOMY?

Discussions of language and concepts usually begin with a chicken-egg problem: which came first, the concepts or the words? On one hand is the notion

that concepts emerge from some nonlinguistic origin. On the other is the claim that they are created, or at least heavily influenced, by one's native language. There are several reasons to reject this dichotomy from the outset. First, even a cursory analysis of the contents of number concepts indicates that there can be no clear leader. So many verbal and nonverbal components are acquired that one side could not completely precede the other. Second, although these extreme positions are often used to frame research, no one lays claim to either one. Despite ongoing debate about the developmental origins of number, modern investigators agree that number concepts are ultimately a mix of verbal and nonverbal components and that there is a bidirectional influence between these. Finally, presenting the issues in such gross caricature does an injustice to early theorists who, despite being strongly associated with certain ideas, also viewed these interactions as complex and bidirectional. Nonetheless, these two extreme views have framed much of the research related to number words and number concepts. For that reason, we begin with a closer examination of the distinction itself and its inherent problems.

1. Concepts Lead Language

The idea that concepts have nonlinguistic origins seems so intuitive that it can be difficult to imagine an alternative. After all, words are arbitrary symbols that derive meaning only by mapping onto a referent. The word "dog" does not mean anything until it is mapped onto an instantiation of a dog. This suggests that we must first develop some understanding of "dog" from nonlinguistic experience—perhaps guided by innate learning mechanisms or sensitivities. On this view, language is the icing on the cake—a means to communicate with others about the many ideas accumulating in our nonverbal stockpile (Fodor, 1983; Pinker, 1994).

Arguments to this effect have been made throughout the history of research on number development (Russell, 1919; Piaget, 1965; Beilin & Kagan, 1969; Beilin, 1975). For example, Piaget claimed that early number concepts emerge from the synthesis of two logical concepts, namely, class and ordinal seriation. Because early counting is initially a rote procedure, and because children fail to demonstrate logical reasoning even after they have mastered conventional counting, Piaget rejected the notion that number language contributes significantly to this development. Russell (1919) held a similar position and argued that early number instruction should be based on logical classes and not on the counting procedure. The basic idea was that because understanding number required more than counting, the origins of such concepts must be nonverbal.

This general position has been revived more recently, but ironically, it is based on evidence that numerical understandings emerge *prior* to conventional counting. For example, some have claimed that humans are endowed with

a prelinguistic core of conceptual knowledge for number based on evidence that infants can detect changes in set size (Antell & Keating, 1983; Starkey & Cooper, 1980; Starkey, Spelke, & Gelman, 1990; Strauss & Curtis, 1981; Xu & Spelke, 2000) and anticipate the results of simple transformations (Simon, Hespos, & Rochat, 1995; Wynn, 1992). A weaker claim has been that certain quantitative skills can develop without mastery of conventional skills, not as part of an innate endowment, but via early experience (Huttenlocher, Jordan, & Levine, 1994; Mix, Huttenlocher, & Levine, 2002b; Saxe, 1988, 1991). Thus, the idea that numerical insight develops without mastery of conventional symbols (i.e., concepts lead language) has played a major role in theories of number development.

2. Language Leads Concepts

In contrast, other theorists have argued the opposite—that language supports the development of certain ideas and skills that might not exist otherwise. Effects of language on cognition are often discussed in terms of the Sapir-Whorf hypothesis: the idea that our concepts, and even our perceptions, can be altered by the way our native language parses the world (Whorf, 1956). Numerous studies demonstrating cross-linguistic effects on reasoning and categorization lend support to this claim (e.g., Choi & Bowerman, 1991; Levinson, 1994; Lucy, 1992). Similar evidence has been garnered in the domain of number. For example, Japanese children, whose language has an explicit base-ten structure, demonstrate a better understanding of other base-ten representations, such as place value blocks and written numerals, than their English-speaking peers (Miura & Okamoto, 1989). Here, the structure of the counting system appears to influence how children perceive other situations, such as the relations among place value blocks.

Another take on this position is that cultural tools, such as language, scaffold human thought so that new insights can be gained. For example, much of Vygotsky's work was aimed at testing whether people of different ages could use external symbols to perform cognitive tasks at increasing levels of abstraction (Vygotsky, 1962). There certainly are examples of this type of "tool use" in the literature on number concepts. From an historical perspective, the advent of improved enumeration systems has preceded new conceptual insights. For example, the shift from Roman to Arabic numerals allowed people of the middle ages to invent computations, such as long multiplication (Menninger, 1958). Similarly, some number systems better prepare children for learning computational procedures than others. Fuson and Kwon (1992) found, for example, that Korean children have an easier time learning to solve multi-digit addition and subtraction problems than English-speaking children. Here, the transparent counting system acts as a tool that children can use to gain insight into an unfamiliar computational procedure. As these examples illustrate, the claim that

number words influence number concepts and mathematical thought (i.e., language leads concepts) also has a strong tradition in theories of numerical development.

3. A Case in Point

This chicken-egg dichotomy crystallized in a well-known debate on the origins of number: Principles Before vs. Principles After. The "principles" in this debate refer to the counting principles outlined by Gelman and Gallistel (1978) (see Table I). These principles were not themselves under debate—everyone agreed that they were needed for accurate and meaningful object counting (enumeration). The debate centered on when understanding of the counting principles appeared relative to acquisition of the verbal counting system.

Advocates of the principles-before view argued that children understand the counting principles before they have learned to count. This was possible because, they believed, the counting principles were embodied in both verbal and nonverbal (innate) enumeration procedures. The nonverbal counting principles were considered a skeletal structure that children fleshed out with the details of their culture-specific counting system (Gallistel & Gelman, 1992; Gelman & Meck, 1983). So, much like universal grammar is thought to aid in language acquisition, access to preverbal counting principles was thought to precede and aid learning to count.

The main evidence for this view was that children adhere to the counting principles even before they possess the skill needed to demonstrate these principles through accurate counts. For example, novice counters often use

TABLE I
Counting Principles (adapted from Gelman & Gallistel, 1978)

One-to-one principle	Every item in a display should be tagged with one and only one unique counting tag
Stable order principle	Counting tags must maintain a consistent sequence
Cardinality principle	The final tag used in a count represents the total numerosity of the set
Abstraction principle	Any combination of discrete entities can be counted
Order irrelevance principle	Items in a set may be tagged in any order as long as the other counting principles are not violated

idiosyncratic lists (e.g., 1-3-7-5) rather than the actual counting sequence. Even so, these children seem to act in accordance with the "how to count" principles. That is, those who count "1-3-7-5" use the same stable (albeit incorrect) order on every count (Gelman & Gallistel, 1978). Pre-counters also detected violations of the one-to-one and stable order principles in someone else's counting, even when the set sizes were far greater than those they could accurately count themselves (Gelman & Meck, 1983).

Proponents of the principles-after view did not believe children could access counting principles preverbally. Instead, they argued that these principles are abstracted through experience with the counting routine (Briars & Siegler, 1984; Fuson, 1988). Evidence for improvement in these skills was taken as support for the principles-after position. For example, Fuson found that early adherence to the counting principles broke down when more challenging tasks were used, such as counting large or randomly arranged sets. In contrast to Gelman and Meck's (1983) results, Briars and Siegler found an age effect for detecting counting principle violations, such that 3-year-olds were less likely to object to one-to-one and stable order errors than were 4- and 5-year-olds. Thus, children's understanding of the counting principles apparently improved as their counting skills improved. Also, other studies contradicted the principles-before findings. In particular, several investigators reported that children's own counting was actually more accurate than their error detection, indicating the reverse of the order of acquisition reported by Gelman and Meck (Baroody, 1984, 1993; Briars & Siegler, 1984; Frye *et al.*, 1989).

The Principles Before-Principles After debate is a prime example of the way language and concepts have been polarized in research on number development. But as we have seen, this dichotomy reaches far beyond this debate. Indeed, it permeates much of the research in this area. Of course, few theorists, including those cited here, have taken either position in the extreme. Still, they have been willing to argue strongly for one contribution over the other. And as with other developmental dichotomies (e.g., nature vs. nurture), the change from a categorical division to a continuum merely obscures the polarization of the two extremes. It shifts the question from "All or none?" to "Mostly this or mostly that?" In Section II.B, we consider the problems with even this type of division.

B. PROBLEMS WITH POLARIZATION

Polarizing language and concepts has limited our understanding of number development in two ways. One is a problem of definition. To show an influence of language on concepts (or vice versa), researchers must define "having language" and "having concepts." This critical task is neither straightforward nor simple. For example, which of the many verbal components of number outlined previously would qualify as the definitive test for "having number language?"

Aren't they all necessary? Yet, long before children have mastered every verbal component of number, individual verbal skills may influence conceptual growth.

Moreover, the relation between language and concepts can shift easily depending upon where investigators draw the line between having one or the other. Take, for example, Piaget's research on number conservation. Piaget defined "language" as how high a child could count, and "concepts" as the ability to judge equivalence in the face of irrelevant transformations (e.g., line length). He found that children accurately counted large sets for some time before they conserved number. In fact, he demonstrated that asking children to count the two arrays in the conservation task did not lead to improved performance.

But what if Piaget had defined "language" differently? Using counting to compare sets requires more than simply enumerating the sets accurately. One also needs to know how counting determines cardinality (i.e., knowing that a collection counted "1-2-3-4" has four items)—an understanding that is not achieved until relatively late (Wynn, 1990, 1992). There is also evidence that to conserve number, children must relate counting to ordinality (i.e., understand that $N + 1 > N$) (Baroody & White, 1983; Schaeffer, Eggleston, & Scott, 1974). These examples illustrate that by changing the definition of "having number language," a link between counting and conservation becomes more plausible.

Disagreement about how to define "counting" and "principles" also fueled the Principles Before-Principles After debate. If having principles means demonstrating any adherence to them, then young children appear to have principles. If having principles means demonstrating them under a range of complex and challenging tasks, then principles appear to develop slowly. The fact is that counting experience and counting principles cannot be completely separated—especially not using conventional counting tasks. By the time children can perform any of those tasks, they have had years of exposure to conventional counting. So, no matter how inaccurate their own counts might be, their ability to follow some procedures and detect errors may still grow out of their limited exposure to conventional counting. At the same time, conventional counting tasks do not directly test the claim that children have access to a nonverbal counting procedure. It is theoretically possible for children to quantify sets in accordance with the counting principles even before they have been exposed to conventional counting, but this possibility cannot be assessed using conventional counting tasks. In short, the effects of language appear to come and go depending on what definitions and associated measures are used.

This issue of shifting definitions is, at its core, a case of competence vs. performance—the idea that what one knows can be separated from what one does. The competence-performance distinction is what underlies the claim that one counting task is more valid than another. It is what allows us to debate which conditions reveal what children "really know" about number. And as investigators have debated the validity of different number tasks, the relation

between language and concepts has been pushed back and forth. For example, researchers subsequent to Piaget argued that the number conservation task was not valid because, among other things, it used large sets. In modified tasks using smaller sets, 3- and 4-year-olds demonstrated the ability to judge the equivalence of sets despite spatial transformations (Gelman, 1972). Clearly, children at this age are less skilled counters than children who could pass Piaget's version of the conservation task; a fact that implies counting may have even less to do with conservation than previously argued. However, most preschool children have some understanding of the small number words, even though they are not proficient counters (Wynn, 1990, 1992). So if we shift the language criterion from counting sets to labeling sets, we should find another reversal—language could well lead concepts again.

If everyone could agree on the quintessential measures of both language and conceptual competence, it would be easy to test which comes first. But of course, there is no such thing as the quintessential measure of competence—there is only performance on different tasks (see Mix (2002), Sophian (1997), Thelen and Smith (1994) for discussions). Competence is inextricably connected to this performance. It cannot be separated in any meaningful sense. Thus, for the same reasons that some investigators have rejected the competence–performance distinction in general, we should question a clear unidirectional influence of either language competence or conceptual competence in number development. The appearance of such an ordering is almost certainly an artifact of the particular definitions that were used.

A second problem with polarizing these positions is that to do so requires committing to a particular time scale of analysis—a commitment that is usually not made explicitly, but yet has profound implications regarding how the interactions between language and concepts are characterized. For example, in Mix's work on numerical equivalence, a rather broad time scale was implicitly adopted. Mix and colleagues found that children could match equivalent sets (where one set was hidden) before they demonstrated the ability to count or label the same set sizes (Mix, 1999a,b, 2004a; Mix, Huttenlocher, & Levine, 1996). They then concluded that a nonverbal representation of small sets likely precedes verbal counting. The underlying assumption was that, because children could not use conventional counting to mediate their comparisons, they must use a nonverbal process instead. The time scale adopted in this research was 6-month increments—the difference between one age group tested and another. Thus, the further (implicit) assumption was that language and concepts probably did not interact until children were better counters, after which they began to recognize more abstract numerical comparisons. Although this latter claim specified a bidirectional influence between the nonverbal, high-similarity comparisons and verbal counting, this influence appeared to take place on a scale of months or even years.

However, language and concepts likely interact on a much closer time scale. For example, when a child at play hears two sets labeled with the same count word, this brief input could cause an attentional shift—one that could temporarily support a numerical comparison. Many such interactions could take place well before children can produce the labels themselves in an experimental task. Thus, what one claims about the influence of language on concepts, or lack thereof, is intimately tied to what time scale one chooses. And most existing research on number concepts uses time scales too broad to capture any subtle interplay between the two. (See Thelen and Smith (1994), for further discussion of time scales in developmental research.)

C. CURRENT CONCEPTUALIZATIONS

Given the problems with polarizing language and concepts, it is natural to ask, why can't it be both? And the answer is that, of course, it is both—a mixture of nonverbal and verbal influences that promote cognitive growth through their interactions. Current conceptualizations of the relation between number words and number concepts take this middle ground. Rather than strongly emphasizing one contribution over the other, these views describe an alternation between verbal and nonverbal influences, differing mainly in how to characterize the nonverbal component. Thus, instead of posing a chicken–egg problem, these views take more of a seesaw approach. However, because a dichotomy between verbal and nonverbal contributions remains, many of the same problems remain as well.

1. *Language Transcends the Limits of Innate Knowledge Structures*

In one class of current conceptualizations, investigators assume that humans are innately endowed with core knowledge systems for number. Although these systems are seen as an important foundation for mature number concepts, the argument is that they have significant limitations. Language is portrayed as the catalyst that allows young learners to transcend the limitations of their innate systems and create more powerful knowledge structures (Carey, 2001; Gelman, 1991; Spelke & Tsivkin, 2001; Wynn, 1998).

In one of the best articulated accounts, Spelke started with the assumption that human infants and many nonhuman animals possess two separate systems for representing number nonverbally (Spelke, 2003; Spelke & Tsivkin, 2001). One system represents items exactly but only works for small numbers. It uses a tracking mechanism that assigns a mental token to each object in a group. These tokens function as pointers to the objects' locations. Because there is a one-to-one relation between tokens and objects, the set of tokens can be used to represent the exact number of objects. However, only a few pointers can be active at any one time due to constraints on selective attention. Furthermore, although

the representation preserves the individuality of the objects, it does not provide a representation of the whole group (i.e., in the way that a number word like “three” verbally represents a set’s cardinality).

The other system represents large sets but only approximately. It is based on the accumulator mechanism, proposed by Meck and Church (1983) to explain timing and counting in rats. This mechanism works by emitting pulses of energy at a constant rate. As items are tagged, these pulses are gated into an accumulator. The resulting fullness of the accumulator, or its magnitude, provides a representation of number. However, it is inherently inexact, even for small sets, because there is not a one-to-one relation between pulses and items. Also, in contrast to the exact system, this representation does not preserve the individuality of the items, though it does represent the group as a whole. Thus, both systems have inherent limitations—the first system being limited to set sizes that the object tracking mechanism can handle (i.e., <4) and the other being limited to rough estimates. Only verbal humans, Spelke (Spelke, 2003; Spelke & Tsivkin, 2001) has asserted, can represent all set sizes exactly and they do so by counting.

An important aspect of Spelke’s conceptualization is that the two core knowledge systems for number are independent of each other and highly encapsulated. That is, though they both represent an aspect of number, they do not interact so as to provide the basis for a complete number concept (i.e., the ability to represent a collection composed of individual items). This means that when children encounter a small set, they should produce two representations of it—one approximate and one exact—without seeing that these representations are related. “By our hypothesis, the child has two systems for representing arrays containing [for example] two objects... Because of the modularity of initial knowledge systems, these representations are independent. When young children hear the word *two*, therefore, they have two distinct representations to which the word could map and no expectation that the word will map to both of them” (Spelke & Tsivkin, 2001, p. 85).

The critical question in this model, then, is how children see that these systems are related and, thereby, overcome the inherent limitations of each. The answer, according to Spelke, is via acquisition of number language. Number language serves to conjoin these two representations because it provides (a) a domain general medium that allows domain specific knowledge to co-mingle and (b) a format that invites the combination of distinct concepts or systems. In Spelke’s account, children make sense of counting by seeing that the same small number words map to both preverbal representations. “... because the words for small numbers map to representations in both the small-number system and the large-number system, learning these words may indicate to the child that these two sets of representations pick out a common set of entities, whose properties are the union of those picked out by each system alone” (Spelke & Tsivkin, 2001, p. 85). Having made this important inference, children are in a position

to generalize to larger sets because the low end of the counting sequence (i.e., “one–two–three”) is connected to the higher end that refers to larger sets.

Now, let us return to the larger question of how language interacts with concepts in terms of the “language first—concepts first” dichotomy. We see both positions represented in Spelke’s account. Initially, preverbal representations precede and act as referents for the first few number words (concepts first). However, once these initial mappings have been carried out and conventional number language has been integrated with the preverbal representations, children are poised to achieve significant conceptual gains (language first). Thus, Spelke’s account involves a bidirectional influence between language and concepts. Yet it achieves this in only a very rough sense.

In a similar vein, Carey (2001) distinguished between two possible relations involving number language and number concepts—a distinction that bears strong resemblance to the polarization outlined in the previous sections. The first relation, dubbed the “continuity hypothesis,” holds that verbal structures are isomorphic to preverbal structures and, therefore, involve no conceptual change when they are acquired—it is a simple mapping of words to pre-existing concepts. The alternative relation, attributed to Whorf, is the idea that some preverbal concepts are incommensurable with the structure of the relevant language. These cases involve dramatic conceptual change that takes place as language is acquired. Carey argued that processes based on both relations underlie numerical development, and we can determine which situations involve which processes by evaluating whether competence is exhibited in prelinguistic babies or only in children and adults with language.

To this end, Carey (2001) outlined five aspects of number that are reflected in human language, such as singular vs. plural and the count–mass distinction, and reviewed the existing literature to determine which of the five are understood by prelinguistic infants. She concluded that even very young infants comprehend four of the five components of number (see Table II). Thus, these notions would seem to develop without language and could, therefore, serve as conceptual referents for the relevant grammatical structures as language is acquired. However, the fifth relation, integer representation, was not evident in prelinguistic infants. Carey reasoned that language input would be required for integer representation because neither of the systems that have been proposed for representing number nonverbally (i.e., the exact system for small number or the approximate system for large numbers) is structured like conventional counting. In particular, neither system can represent cardinality and thus, neither could support an easy mapping between number words and referents. In this way, Carey explained the protracted course by which children bring meaning to the number words and verbal counting.

Although these accounts vary in their treatment of certain points, they share several key assumptions: (a) number development involves a bidirectional

TABLE II

Five Aspects of Number Reflected in Natural Language (adapted from Carey, 2001)

Concept	Example	Evident in infants?
Object individuation	"I remember the toy duck that is hidden under the table"	YES
One vs. another	"I see a duck over here and a duck over there and know they are not the same duck"	YES
Count vs. mass	"Two ducks are distinct individuals, but two piles of sand are just some stuff"	YES
Sortals (nouns) vs. predicates (adjectives)	"Where the duck moves tells me it's an individual, but its color and size do not"	YES
Symbolic representation of integers (i.e., counting)	"One—two—three—four—five. Five ducks"	NO

interaction between verbal and nonverbal structures; (b) the nonverbal structures are what prelinguistic infants and nonhuman animals use; (c) these nonverbal structures play an important role, but they are limited; and (d) what helps children transcend these limitations is acquisition of number language. At some level, these seem like reasonable assumptions. However, on closer examination, we find reason to question them.

First, there is widespread agreement that number development involves an interaction between verbal and nonverbal structures. But the devil (or maybe God) is in the details—details that are left largely unspecified in these accounts. The critical turning point in each of them is when children manage to map small number words, as labels, onto their pre-existing representations for the corresponding quantities. But how, exactly, do children achieve this crucial step? Cross-sectional research indicates that this is an elusive and protracted mapping. Therefore, the specifics of how it is achieved are neither obvious nor likely to be straightforward.

This is a case where time scale may be critical. The accounts described here suggest a long period of stability, during which infants and toddlers use their innate representations, followed by an unspecified mapping process, and then another long—perhaps indefinite—period during which concepts have been transformed. Presented as such, the interactions between language and concepts resemble two large scale, unidirectional shifts that take place in sequence more than they do ongoing bidirectional, bootstrapping. Yet, as we will see, when development is studied on a different time scale, the interactions between number words and number concepts appear much more fluid and tightly linked in time than these accounts imply.

There also are problems with the way these accounts define verbal and nonverbal components. As noted previously, drawing this distinction can be complex and arbitrary. However, these accounts draw a clear line between what seemingly nonlinguistic beings (i.e., infants and animals) know and what those of us with language know. But at what point do we say that infants shift from prelinguistic to linguistic? When they have been exposed to words? When they comprehend words? When they begin to speak themselves? It seems difficult to say with certainty that any of infants' sensitivities are based on purely nonverbal information, when humans are immersed in linguistic input beginning prenatally. And what about the opposite end—the concepts of those who have acquired language? The demarcation adopted here implies that *only* prelinguistic infants possess nonverbal thoughts, even though cognition is likely a mixture of verbal and nonverbal components, even in language experts (i.e., adults). Indeed, this was one of Vygotsky's claims—that although language-mediated thought increases with development, it never fully eclipses nonverbal thought.

Finally, there is cause to question these accounts on empirical grounds. Both assume that the nonverbal foundation of early number concepts is comprised of abstract representations generated by two innate processes (i.e., the small exact number system and large approximate number system). In support of this, Spelke and Tsivkin (2001) cited evidence for dissociable enumeration systems in adults. For example, they noted that adults with acalculia, who cannot provide exact solutions to arithmetic problems, often give estimates of correct answers. This certainly suggests that humans have different systems for representing number (though this has been recognized for some time: Jensen, Reese, & Reese, 1950; Jevons, 1871; Kaufman *et al.*, 1949; Taves, 1941). And it may show that these systems are localized in different parts of the brain. However, it does not indicate that either of these systems are innate, or even early emerging.

In fact, research involving young infants has yet to clearly demonstrate any sensitivity to discrete number, with or without implicating either proposed representation (see Mix, Huttenlocher, and Levine (2002a,b), for a discussion). In brief, the evidence cited in support of numerical sensitivity in infants is undermined by confounds with non-numerical cues. For example, habituation studies showing that infants can discriminate between different set sizes (e.g., Antell & Keating, 1983; Starkey & Cooper, 1980; Starkey, Spelke, & Gelman, 1990; Xu & Spelke, 2000) are undermined because number was allowed to covary with contour length and/or area. When these variables have been separated, infants fail to respond to number, but continue to respond to changes in non-numerical cues (Clearfield & Mix, 1999, 2001; Feigenson, Carey, & Spelke, 2002). This casts doubt on the idea that either representation provides a nonverbal referent for the number words, because it is currently unclear when or how such representations develop.

If future research succeeds in demonstrating that these representations emerge without exposure to language, by testing very young infants under conditions that strip away or randomize every other quantitative cue, this still might not bear on what infants do in everyday situations where these other cues are available (Mix, Huttenlocher, & Levine, 2002a). This is a problem for accounts that assume children map number words onto nonverbal, abstract representations. If children use non-numerical information when it is available, then why would they map the number words to abstract representations of number and not to this information? For example, if a one-year-old hears his mother say, "two cookies," what tells him to map this phrase to a mental representation of the cookies' two locations, rather than the amount of cookie relative to the plate? For that matter, why would he map the number word to a mental representation at all, whether in terms of number or amount, when the actual objects are right in front of him?

2. *Language Transcends the Limits of Experiential Knowledge Structures*

An alternative to the innate knowledge models is the view that nonverbal representations of number develop in early childhood rather than comprising an innate endowment (Huttenlocher, Jordan, & Levine, 1994; Mix, Huttenlocher, & Levine, 1996, 2002a). Huttenlocher *et al.* (1994) proposed that young children develop a symbolic representation of exact number, or a mental model. In this representation, children create an array of imagined entities that stands for each of the actual entities in the real array. For example, children would represent a cookie, a brownie, and a croissant with an imagined cookie, an imagined brownie, and an imagined croissant. Huttenlocher *et al.* remained agnostic with respect to how much detail is actually preserved in these models. That is, the representations could consist of rich images of each object or they could be as sparse as a pointer. The main idea was that number is incidentally preserved because there is a one-to-one relation between the actual entities and their symbols. Huttenlocher *et al.* did not claim that the mental model was the earliest representation of quantity and allowed for the possibility that some sensitivity to quantity might be present in infancy. However, contrary to the innate knowledge views, the mental models view assumed that infants' representations are approximate—even if they are based on discrete individuals. Thus, the emergence of a mental model was considered significant because it constituted the earliest representation of *exact* number available to humans.

These claims were based on a series of experiments on young children's calculation ability. Huttenlocher *et al.* (1994) used a nonverbal task in which addition and subtraction problems were presented using concrete objects. For example, to present the problem $1 + 2$, children were shown a single block for a few seconds before it was hidden beneath a cover. Next, two more blocks were shown sliding under the cover. The child's task was to produce a set of blocks equivalent to the resulting set, even though this remained hidden from view.

Some children were able to solve such problems as early as 30 months of age. This is years before they receive instruction on the conventional algorithms for addition and subtraction, and so, not surprisingly, it is also years before they demonstrate competence on analogous problems presented in a verbal format (i.e., as word or number fact problems) (Levine, Jordan, & Huttenlocher, 1992). Thus, these authors concluded that children were using a nonverbal process to represent the entities and arrive at a solution.

These experiments also documented a shift from inexact to exact responses in the nonverbal calculation task. As noted previously, exactly correct responses were observed by 30–36 months of age. However, younger children (24- to 30-month-olds) did not perform randomly. Their responses were approximately correct. Thus, these youngsters understood that adding should result in more and subtracting should result in less, and they were able to estimate the number of items to a certain degree of accuracy, but did not reach solutions with the precision exhibited by slightly older children. Huttenlocher *et al.* (1994) argued that children who produced approximations of the correct answer did not yet possess a mental model. This shift to exact responding was considered significant because it revealed an intermediary stage between the approximate quantitative sense of infants and the advent of verbal counting. Huttenlocher *et al.* speculated that further development, including the ability to deal with larger sets, would require mastery of conventional skills,

Indirect evidence for this second shift, from limited but exact nonverbal representations to more powerful verbal representations of number, was provided in Mix's studies of numerical equivalence (Mix, 1999a,b, 2004a; Mix, Huttenlocher, & Levine, 1996). In these studies, 3- and 4-year-olds completed a forced choice matching task in which they chose a set of dots that was numerically equivalent to a standard. Across experiments, the contents of the standard sets were varied, thereby varying the degree of similarity between the two matching sets. As we discussed previously, when the standard and the choice sets were highly similar (e.g., all black dots), even children with little or no conventional counting skill matched them correctly. Although the matching sets in this condition were similar in many aspects besides number, accurate performance required numerical reasoning because the distractor sets shared the same object-based similarities as the matching sets. Also, the standard set was hidden when the choice cards were revealed; so children had to mentally represent the number of objects. That young children could do so without the counting skills needed to represent the sets verbally was further evidence that they possessed something like a mental model.

However, it appeared that this nonverbal representation was limited in that children could not perform more abstract comparisons using it alone. Only children who were proficient counters also recognized equivalence between more disparate sets (e.g., sounds and black dots). Mix and colleagues speculated that

this pattern might reflect a deep relation between language and concepts—one in which the number words organize attention like other category labels (Mix, 1999a, 2004a; Mix, Huttenlocher, & Levine, 2002b; Sandhofer & Mix, 2003). For example, research outside the domain of number indicates that words alert children to possible commonalities between items and help to focus their attention on shared dimensions (Gelman & Markman, 1987; Gentner *et al.*, 1995; Sandhofer & Smith, 1999; Smith, 1993; Waxman & Markow, 1995). We have argued that number words serve the same purpose. That is, hearing two sets named with the same count word could prompt a comparison process that leads to recognition of number as a dimension of similarity. Mix, Huttenlocher, and Levine (2002b) also argued that language acts as a placeholder, or memory aid, as children acquire new skills. For example, children can use the number words to stand for the numerosity of a hidden set while they choose an equivalent set, rather than carrying out the potentially laborious process of comparing the represented (hidden) objects and visible objects one to one.

There are parallels between these accounts and the innate knowledge accounts. And these parallels lead to some of the same problems we have already outlined. The main drawback remains reaching a satisfactory separation between verbal and nonverbal contributions. In the innate knowledge models, nonverbal was defined as whatever prelinguistic infants know. In the present models, the line is drawn much closer to the acquisition of conventional skills. Therefore, what counts as verbal varies from task to task. Comparisons are made between groups of children who demonstrate mastery of the conventional skills (related to a particular task) and those who do not. When conceptual competence is revealed in children lacking these skills, it is attributed to a nonverbal process.

It is important to know which tasks require conventional skills. However, demonstrating that children do not use conventional algorithms does not necessarily mean they are using a purely nonverbal process instead. We know that by $2\frac{1}{2}$ years, the age at which children begin to solve nonverbal calculation problems exactly, they have been exposed to conventional counting for many months, have number words in their vocabularies, exhibit rudimentary counting skills, and may understand the meanings of “one” and “two” (Fuson, 1988; Gelman & Gallistel, 1978; Wynn, 1990, 1992). So, children’s performance on the nonverbal calculation task could have been mediated to some extent by these words. Perhaps children represented the hidden sets with a number word or computed the solutions by counting. Huttenlocher *et al.*’s (Huttenlocher, Jordan, & Levine’s, 1994) finding that children first demonstrate competence on small number problems (e.g., $1 + 1$) is consistent with the fact that children at this age may know only the meanings of “one” and “two.” Maybe the shift from approximate to exact responses reflects improved number recognition or counting ability.

The same criticism could be leveled at Mix’s work on numerical equivalence. The criterion for counting competence used in these studies has been the Give-a-Number task (Wynn, 1990) in which children are asked to produce sets of various numerosities (e.g., “Give me three blocks”). However, this is a high-level test of numerical understanding because it requires children to create different set sizes rather than simply recognize or label them (Benson & Baroody, 2003). This is akin to assessing children’s understanding of the word “dog” by asking them to draw a picture of one. When children fail to match equivalent sets until they achieve this understanding of number words, it provides strong evidence that conventional skills are needed. However, when children match high-similarity sets *without* having passed the Give-a-Number task, it does not necessarily mean they relied on a purely nonverbal representation. They could, instead, rely on partial knowledge of the conventional number words—a level of mastery that is sufficient to support simple comparisons but not less obvious matches (i.e., those with less perceptual support).

A second problem is that the interactions between verbal and nonverbal concepts described in the experiential knowledge models are still rather broad and unidirectional. Like the innate knowledge models, these models describe a series of three, rather sweeping interactions: (a) nonverbal representations of small numbers develop; (b) number words map onto these representations; (c) children gain insights that support new concepts. These interactions are refined, somewhat, by the hypothesis that they are driven by the same well known processes that drive language acquisition and categorization in other domains. However, they are still painted in broad strokes—at a grain of detail that does not reflect the way these interactions likely unfold in real time.

3. Language and Concepts Develop Simultaneously

The models we have reviewed so far have polarized number language and number concepts by either emphasizing the importance of one over the other or describing broad, seesaw interactions between the two. We have pointed out that such polarization has inherent problems—problems that have hindered research progress on this topic. However, other models have attempted to capture the interplay of verbal and nonverbal processes at a more detailed level (Baroody, 1992; Canobi, 2004; Fuson, 1988; Rittle-Johnson & Siegler, 1998). These models provide a framework for thinking about the bootstrapping of words and concepts in real developmental time.

To illustrate, consider the iterative model (Baroody, 1992; Baroody & Ginsburg, 1986). In this view, numerical development is a gradual incremental process, in which incomplete knowledge repeatedly combines with new input to support new inferences and procedures. The model is built upon Anderson’s (1984) distinction between weak and strong schemas, which held that development proceeds from disconnected, task-specific, and logically incoherent

knowledge structures (weak schemas) to well-connected, highly generalizable, and logically coherent knowledge structures (strong schemas). Children are hypothesized to move along this continuum by repeatedly bootstrapping among various conventional skills and underlying concepts. The iterative model was agnostic regarding innate origins of number representation. Instead, its focus was on the developmental process that might underlie changes from infancy to school age, with or without an innate contribution.

Several key aspects of this point of view relate to the issues we have raised so far. For one, it holds that conventional skills, such as counting, can be acquired piecemeal and initially without meaning, through imitation, practice, and reinforcement. Thus, children could be exposed to number language and develop some mastery of it without completely understanding it. Importantly, however, these partial understandings were considered useful, indeed crucial, contributions to children's learning.

For example, children with partial understanding of counting might tag items with an idiosyncratic count word sequence (e.g., "five-three-two-three"). And they may not understand the implications of counting for determining cardinality, equivalence, and so forth. But as long as they know enough to say one word for each object, their counting attempts could generate important data for them. The fact that this idiosyncratic list might "fit" two different sets in terms of one-one correspondence could signal that the sets were the same. This signal could inspire further exploration of similarity between the sets. By exploring the ways in which these sets were similar, children might broaden their ideas about numerical equivalence classes. Such interactions are the essence of a genuine bidirectional influence between language and concepts—one in which number words and number concepts feed on each other at every point in development, no matter how limited and immature each side may be.

A further implication of the iterative model is that the earliest interactions between number words and concepts should be couched in specific contexts. This is because weak schemas—the starting point for development—should be task-specific and disconnected from each other. If children acquire initial skills through imitation and reinforcement, then these early contexts should follow from social routines or recurring situations in which parents have remarked on number. For example, Baroody (1992) observed that many children learn number words in the context of their age. They learn to say "two" or "three" when asked how old they are. Two- and three-year-olds certainly do not grasp the concept of years, so this is, at least initially, a meaningless mapping. But even this rote mapping could scaffold a child into deeper understanding under the right circumstances (e.g., learning to hold up the correct number of fingers at the same time, which could in turn provide a concrete referent for future mapping). These observations are significant because they demonstrate how first skills could emerge in parent-reinforced, social routines. From this, it follows that a major

developmental trend will be the gradual decontextualization in which both skills and concepts become less encapsulated by these routines.

Note that this is a very different sense of decontextualization than the one discussed by Spelke (Spelke, 2003; Spelke & Tsivkin, 2001). In her view, it is the two mental representations of number that are initially encapsulated. Although these representations can operate across a variety of contexts, and in that sense already are decontextualized, they are encapsulated because they function independently, as disconnected thought processes. In simultaneous or iterative models, it is numerical competence or knowledge that is initially encapsulated because it is embedded in specific contexts and routines. Here, the process of decontextualization involves generalizing over these disparate situations.

4. Conclusions

Most current models of number development fail to capture the complex interactions between verbal and nonverbal processes. There is no definitive place to draw the line between verbal and nonverbal, so regardless of where these models have drawn it, they are probably not correct. Because they view developmental change on such long time scales, verbal and nonverbal components meet only in broad, unidirectional passes. In contrast, Baroody (1992) and others (Canobi, 2004; Rittle-Johnson & Siegler, 1998) have taken an approach that emphasizes fine-grained, bidirectional interactions. From this point of view, development involves a complex scaffolding of partial knowledge and skills—verbal and nonverbal interacting context-to-context, moment-to-moment. We do not mean to imply that other models are in opposition with these ideas. Indeed, all of the conceptualizations reviewed here are compatible with an iterative or simultaneous model. The difference is that the former, unlike the latter, have not articulated these ideas explicitly. This is not a trivial omission, because it leaves an artificially simplistic impression of the developmental process underlying these interactions.

One reason that other accounts have failed to capture the complexity of number development may be that they are based on data that tends to obscure it. Most existing research on counting, equivalence, and cardinality has used stripped-down experimental tasks presented to large groups of children in cross-sectional designs. Comparing children in 6-month increments leaves the impression that change occurs in broad strokes. When 4-year-olds succeed on a task that $3\frac{1}{2}$ -year-olds fail, it seems as if there is one knowledge state at age $3\frac{1}{2}$ years and a different knowledge state 6 months later. This may be true, but children might pass through many other knowledge states along the way.

Furthermore, testing children with controlled experimental tasks virtually ensures that concepts will seem abstract because abstraction is what these decontextualized situations require. Even when tasks are designed around

naturalistic situations or play activities, they still are imposed by the experimenter. Unless the experimental contexts happen to overlap completely with the contexts in which each individual child has discovered number at home, they will require a certain degree of generalization. Indeed, Mix (2002) argued that these varying degrees of overlap are at the heart of the competence–performance distinction because tasks that tap into informal knowledge at a younger age (via serendipitous overlap with contextualized knowledge) may appear more valid than those that do not. However, naturalistic tasks are not necessarily more valid than artificial tasks. In fact, artificial tasks are the best way to test whether knowledge has been abstracted. The problem is that, on its own, such evidence tells us little about how children got there. To address *this* question, we need to take a different approach—one in which we turn up the microscope, as Thelen and Smith (1994) put it, and study development at close range.

III. Number Development at Close Range

Getting close to the developmental process means moving beyond large scale, cross-sectional research to a more intense focus on the development of individual children. This requires a shift to long-term diary, case study, and microgenetic training designs. It requires one to consider performance both on standardized tasks and in children's spontaneous activity.

There is a rich history of such methods in developmental psychology, beginning with the baby biographies of the 19th century and the bedrock research of early theorists, such as Jean Piaget. More recently, these approaches have yielded fascinating insights in research on language development (e.g., Huttenlocher, Haight, Bryk, Seltzer, & Lyons, 1991; Mervis, 1985). The work of Katherine Nelson, in particular, has provided a wealth of fine-grained observations of language unfolding in the natural environment (e.g., Nelson, 1996). Her work has revealed many of the same patterns that we might expect to find for number development—early event related (contextualized) knowledge structures, idiosyncratic performance based on individual learning histories, and in-the-moment effects of language.

Studies using such methods have periodically appeared in the domain of number for many years. However, only recently has there been a more concentrated effort to track number development at close range. In this section, we review and integrate this research, with an eye toward the interaction of language and concepts in particular. These studies provide evidence for five main themes regarding numerical development: (1) it is contextualized; (2) it is piecemeal; (3) it is socially scaffolded; (4) it differs across individuals; and (5) it uses domain general processes.

A. NUMBER DEVELOPMENT IS CONTEXTUALIZED

The context-specificity of early development can be easy to miss when investigators use standardized experimental tasks, because performance on these tasks requires a certain degree of decontextualization. Children who cannot perform experimental tasks may have partial knowledge, deeply contextualized knowledge, or no knowledge at all. To determine the status of these children, we need to look at the behaviors they generate themselves.

This was the aim of a diary study that Mix (2002, 2004b) completed with her son, Spencer. Observations were recorded from the time Spencer was 12 months old until just after his third birthday. They focused on two main activities: (a) use of one–one correspondence in spontaneous play, and (b) attempts to count and use number words to label sets. In addition to the diary observations, Spencer was tested using standardized numerical equivalence and counting tasks on a monthly basis, beginning at 20 months of age. Competence in both nonverbal and verbal aspects of number was evident very early in Spencer's development. However, in both cases, this competence was highly contextualized.

This contextualization was evident in his understanding of numerical equivalence. For many months early in the study, he experienced one-to-one correspondence in his play—handing out toys to other children, touching objects one by one, or taking turns in simple games and routines. These activities did not require explicit understanding of numerical equivalence, but likely provided important data for the development of such an understanding. At 20 months, he spontaneously produced a set of objects that was equal in number to another set hidden from view. Specifically, he took exactly two dog treats for his two dogs, waiting in another room. This was not a coincidence. Over 3 weeks, he repeatedly performed the same task with almost perfect accuracy using different-sized treats. Yet, he was unable to perform the same task when the context changed. He failed Mix's (1999a,b) forced choice matching task—even for the numerosity “two.” And, he failed on a slight variation on the dog treat task, one in which he was asked to give his toy trains “train treats.” Spencer did not explicitly match sets in any other situations until he was 30 months old, when he went into the backyard and returned with three sticks, one for each person sitting at the dinner table (i.e., his two parents and a dinner guest). Like children in previous cross-sectional studies, Spencer began matching sets in the forced choice equivalence task starting at 34 months, with high-similarity (disks-to-dots) comparisons first. Unfortunately, the diary study concluded soon after that, so we do not know whether spontaneous matches increased around the same time that his notions of numerical equivalence became decontextualized.

During the same developmental period, Spencer's acquisition of small number words was similarly contextualized (Mix, 2004b). His earliest uses of

the number words did not actually reference sets of objects. Instead, he used number words to label written numerals. This began with the numerals that appeared in several of his board books, but he eventually came to recognize numerals on signs, license plates, and addresses as well. At 23 months, he began using number words to label sets of objects. His first mappings were restricted to the number "two" and they always occurred within a particular linguistic frame: "Two ____, One. Two." For about a week, he labeled only sets of shoes using this frame (i.e., "Two shoes. One. Two."). Then he extended to other object sets, including two dogs, two spoons, and two straws, using the same frame. At 24 months, he began using the variant "____, ____, two ____." For example, for two trains, he would say, "Train. Train. Two trains." This frame appeared frequently for the next 6 weeks and, during this period, he did not label sets numerically without using it. Fuson (1988) reported that her daughter Adrienne used the same frame at age 22 months.

Throughout this period, Spencer failed all tests of conventional counting. In the Give-a-Number test, he failed to produce two objects on request and when asked how many objects were in a set of two, he responded with an idiosyncratic string of number words. Thus, although he correctly labeled different sets of two, his use of the number word "two" was far from decontextualized. In fact, it was deeply contextualized in two ways. First, it was initially restricted to specific situations—first labeling numerals, then labeling shoes. Second, these early attempts were embedded in specific linguistic frames. A similar pattern was reported in another diary study that tracked the number development of a young boy, Blake, from 18 to 49 months of age (Benson & Baroody, 2002). Blake's first number word also was "two" and he used it, initially, only when asked his age. His parents had reinforced this response in preparation for his upcoming birthday. Although this was a simple association without cardinal meaning, it is noteworthy that his first use of a number word occurred in this, and only this situation.

Both boys also overgeneralized the response "two" to the question "How many?" That is, regardless of a set's numerosity, when asked informally how many items were there, both boys tended to say, "two." This was true even though both boys spontaneously labeled sets of one and two correctly. The following excerpt from Spencer's diary (Mix, 2004b) illustrates this tension:

(5/24/01; 26 months) While Spencer was taking a bath, I threw in one toy fish. Then I added two frogs, one by one. Spencer remarked, "Oh! Two frogs!" Then I threw in a third frog. Spencer said, "Oh! One frog and two frogs." I asked, "How many frogs all together?" He responded, "Two frogs." I replied, "No, three frogs." A little later, Spencer spontaneously asserted, "Two frogs." I replied, "No, three frogs." He countered, "No, one frog, one frog, one frog, and one fish." I asked, "How many is that?" His response: "Fish need water."

Although Spencer correctly labeled sets of one and two, he insisted that there were two frogs when queried. The fact that both he and Blake spontaneously

generated correct labels for one and two suggests that these overgeneralization errors were specifically embedded in the "How many?" routine. Perhaps Spencer and Blake viewed the question "How many?" as part of a script where the other person always answers, "two," presumably because they had been reinforced for this response when it was correct.

B. NUMBER DEVELOPMENT IS PIECEMEAL

One of the most striking patterns to emerge from diary and longitudinal research is that acquisition of the number words has a distinct stepwise or piecemeal quality (Benson & Baroody, 2002; Fuson, 1988; Mix, 2004b; Sandhofer & Mix, 2003; Wagner & Walters, 1982; Wynn, 1992). For example, when Wynn (1992) tracked children's development using a variety of standardized tasks, she found that the number words were acquired in discrete stages. In the first stage, children can give one object on request and also identify which of two cards shows only one object (mean age = 33 months). Children at this level did not point to a single object when queried with other number words. For example, given a card with one fish vs. a card with two fish, these children never pointed to the card with one fish when asked to point to two. In fact, they consistently inferred that the "not one" option (i.e., the card with multiple items) was the referent of words other than "one." However, this did not reflect knowledge about the specific cardinal meanings of these words, because the same children performed randomly when both cards depicted multiple items (e.g., two vs. three). Thus, Wynn concluded that children first discover the difference between "one" and "more than one." In the second stage (mean age = 35–37 months), children correctly produced and identified sets of one and two, but failed to distinguish among larger numerosities. Within a few more months (mean age = 38–40 months), children correctly identified sets of three as well. Finally, children were able to produce and identify all the sets in their counting range (i.e., four and greater) at about the same time (mean age = 42 months).

Wynn's (1992) study indicates that number words are acquired in a piecemeal fashion. However, as we have already discussed, this development begins much earlier than the ages tapped by her tasks. Diary research demonstrates that children start sorting out the meanings of "one" and "two" in specific contexts around their second birthdays or even earlier (Benson & Baroody, 2002; Fuson, 1988; Mix, 2004b). This is 9 months before children demonstrated an understanding of "one" and 13 months before they demonstrated an understanding of "two" in Wynn's research. Moreover, the diary studies suggest that rather than leading with the number "one," it is the number "two" that may have special status in very early development. All three studies reported that functional use of the word "two" preceded functional use of "one." Also, Spencer labeled

sets of two much more frequently (five times more) than sets of one (Mix, 2004b). Finally, whereas these children usually labeled both "one" and "two" accurately, they tended to overgeneralize the label "two" when queried by adults.

Labels for larger sets (i.e., three and four) appeared in children's spontaneous utterances several months after the appearance of "one" and "two" (Benson & Baroody, 2002; Mix, 2004b). Although some overgeneralization of the word "three" was observed, children were generally accurate from early on (29 months of age in Spencer's case). Perfect functional use of the word "three" was evident before perfect use of the word, "four," but these milestones occurred relatively close in time (within weeks for Spencer). Thus, as Wynn observed, there was a stagelike progression in the emergence of informal mappings for the small number words. However, in contrast to Wynn's findings, the order of emergence was different (i.e., two appeared before one), the separation of the stages was less apparent, especially for one and two, and the ages of acquisition were very discrepant. Specifically, by roughly the same age as the children Wynn (1992) classified as Stage I (i.e., those who demonstrated only an understanding of one vs. more than one), Blake had already demonstrated highly accurate use of both "one" and "two," and Spencer had done so for all the small number words, "one" through "three."

Although it is not entirely clear why children would repeat the same sequence in Wynn's (1992) standardized tasks that they appear to pass through in informal labeling, it seems certain that the standardized tasks did not provide the same scaffolding that children have when they label sets themselves. In other words, we might ask why certain everyday situations seem to draw correct numerical labeling out of children earlier than the Give-a-Number or Point-to-X tasks used by Wynn. The diary data suggest that labeling pairs of observable objects is what gets the ball rolling. As we noted, children label sets of two earliest, most frequently, and with great accuracy. Furthermore, the vast majority of sets they label consist of observable objects (rather than mentally represented or remembered objects) (Mix, 2004b). So, what makes this situation special?

Mix (2004b) speculated that the answer may be a coincidence between limits on children's comparison ability and an initial misinterpretation of the word "two." In the following excerpts, Spencer incorrectly extended his number frame for "two" to larger sets. Such errors were rare, so it was not the case that he routinely mislabeled larger sets "two," as if he took the word to mean "many." Instead, the following anecdotes suggest that for Spencer, the word "two" had more to do with the similarity among items in a set than it did with their cardinal number:

2/10/01 We had a carpet sample board with about 20 carpet squares. Spencer remarked, "Blue!" Then, he slapped the board 5 times, contacting 8 squares, while saying "A blue" with each slap. Then he said, "Two blues."

2/24/01 Spencer pointed at each of the three living room windows and said, "Window, window, window. Two windows."

3/8/01 Spencer saw three apples on the dining room table and said, "Apple. Apple. Apple. Two apples."

Clearly, Spencer was aware of the similarity among items in these sets and acknowledged it, both by tagging each item through touch or pointing, and by labeling each item with the same name. The fact that he summed up these comments with the number word, "two," regardless of the set's actual size, suggests that he misinterpreted "two" to mean "same." This would be a sensible error, given that number words apply to groups of things that share some commonality. Furthermore, there is little in the input to indicate that "two" or any other number word refers to number in particular, rather than something like "again," "another," or "same."

But why use only "two" this way, and not the rest of the number words? One reason might be the relative ease of comparing two items. If it is easier to determine that two objects are the same than it is to evaluate the similarity of three or more items, then a child would be more likely to comment on similarity for sets of two. Once children can make more complex comparisons (i.e., when they begin to see similarity for larger sets), they may refer to this similarity as "two," simply because "same thing" is already a major part of what "two" means to them. Exposure to the homonym "too" might further reinforce this misinterpretation. Young children have no way to know that "two" and "too" are different words. Sentences such as, "Mary wants a cookie, too," might provide additional (erroneous) evidence that "two" means something like "another."

Because young children may conflate "two" and "same," it is difficult to say whether their early uses of "two" refer to cardinal number at all. Although Spencer and Blake used the words "two" and "one" discriminately, this discrimination could have been based on the need to comment on similarity, or lack thereof. That is, there would be no reason to say "same" for a single object. Perhaps that is why children fail to demonstrate an understanding of "two" in Wynn's (1992) tasks even though they use it with great accuracy in these informal labeling situations. Clear evidence for the separation of cardinality and similarity would require discriminate use of different words for multiples, such as "two" and "three"—a development that seems to take an additional 6 months to achieve informally, yet still appears to precede Wynn's Stage II by a considerable margin.

The discrepancy between Wynn's (1992) findings and the diary results is a prime example of why verbal and nonverbal change cannot be separated developmentally. Although it could be argued that children do not really understand "two" until they can perform Wynn's tasks, and in that sense are still nonverbal with respect to number, it is equally true that they do not develop these understandings in a verbal vacuum. Children are clearly experimenting with the number words, usually with success, for many months

before they appear to understand them in more structured tasks. This means that the “nonverbal” referents for specific numerosities may well be shaped or even created by exposure to the number words. These partial mappings, in turn, are likely to scaffold deeper understanding of the number words themselves.

During the same period that children acquire the pieces of verbal cardinality, notions of numerical equivalence also emerge in a stepwise manner. As noted previously, cross-sectional research has indicated that the ability to match equivalent sets begins with high-similarity matches between 30 and 36 months, and gradually extends to more disparate comparisons (Huttenlocher, Jordan, & Levine, 1994; Mix, 1999a,b, 2004a). Children cannot completely ignore object similarity in favor of numerical equivalence for another two years, until 5 years of age (Mix, 2004a). This pattern was also revealed in a longitudinal study of children’s number development (Sandhofer & Mix, 2003). Children were tested once a month from 36 to 54 months of age. They completed several versions of the forced choice number matching task that ranged from high-similarity comparisons (i.e., black disks to black dots) to low similarity comparisons (e.g., puppet jumps to black dots). As in previous cross-sectional research, children did not succeed on the full range of comparisons all at once. Instead, success in the high-similarity conditions always preceded success in the low-similarity conditions. There also were effects of set size. Children matched equivalent sets that were small (1–4) over a year before they matched larger sets (5–8). These patterns were replicated in Spencer’s diary data as well (Mix, 2004b).

Taken together, longitudinal and diary research indicates that partial number competence emerges long before reliable performance on experimental tasks (Baroody, Benson, & Lai, 2003; Benson & Baroody, 2002; Mix, 2002). During this period of growing competence, children gradually gather pieces of both verbal and nonverbal understanding. Although we can attempt to study these components separately using verbal or nonverbal tasks, aspects of the two are always present, tightly intertwined in developmental time.

But does this mean they are integrated in the child’s mind? The answer depends on what is meant by “integrated.” The complete integration of number words, counting, and all the possible instantiations of numerosity is the culmination of numerical development—an apex that is not achieved for several years. Yet, there is likely an ongoing bidirectional influence of partial knowledge across the verbal–nonverbal divide well before this achievement. Are these piecemeal interactions integrated from the child’s perspective? Perhaps, but only within specific contexts. For example, saying “Two shoes. One. Two,” implies an integration of counting and cardinality. Though not the same as having decontextualized, principled knowledge of this relation, this may reflect an

explicit integration within that context, just the same. Alternatively, interactions involving partial understanding may not reflect explicit integration, even though they could be a large part of what bootstraps children into such levels of understanding.

C. NUMBER DEVELOPMENT IS SOCIALLY SCAFFOLDED

In our discussion of contextualization, we pointed out the extent to which children’s early number knowledge is tied to specific situations. We speculated that one reason competence emerged in these contexts and not others was the high similarity between items in those sets. However, another quality shared by many of these situations was a high degree of social scaffolding and reinforcement. In fact, naturalistic observations have revealed time and again that numerical understandings emerge first in social games and routines (Benson & Baroody, 2002; Mix, 2002, 2004b)—a basic fact overlooked by research using standardized tasks. For example, when Spencer succeeded in matching treats to dogs, he was likely imitating a routine he had seen his parents perform every day. This matching activity also was an extension of a socially mediated one-to-one correspondence activity he had been spontaneously performing for months—namely, distributing objects (Mix, 2002). Rather than aligning objects, as in matching teacups to saucers, Spencer’s most frequent one–one activity was handing out objects to people, dolls, or animals. Mix speculated that Spencer was reinforced for this activity by the attention and positive social interactions he experienced as the distributor.

Spencer’s number word learning also occurred within various social routines (Mix, 2004b). For example, a series of conversations about number arose within the daily routine of taking vitamins. Spencer liked the taste of his chewable vitamins and would have eaten more, but he was only allowed to have one per day. Consider the following excerpts:

(3/21/01; 26 months) As Spencer was feeding me toast, he said, “Just one, Mommy,” —the same thing I say when I give him his vitamin.

(3/24/01; 26 months)

KSM: Do you want a vitamin?

S: Yeah

KSM: Okay, here you go. Just one vitamin.

S: No! *Two* vitamins (smiling).

(3/30/01; 26 months) When offered a choice of two vitamins, Spencer tried to take them both. But he smiled and said, “Two. *Two* vitamins.”

(4/6/01; 26 months)

KSM: Would you like a vitamin?

S: Just one!

KSM: Right! Just one vitamin a day.

S: Just two (smiling).
 KSM: No, just one.
 S: No, just two (smiling).

These excerpts illustrate how Spencer's early comments on cardinality were couched in familiar, recurring contexts that highlighted number. This context was not supportive simply because he encountered the same objects. It was a social situation for which talking about number served a function—it might get him another vitamin and, failing that, it might make his mother laugh.

Similar contextual support was provided in Spencer's acquisition of the counting sequence. Starting at 23 months of age, he frequently participated in a counting, turn-taking game with his father. For example, if his father said, "One," Spencer would respond "Two." His father would then say, "Three," and Spencer would reply, "Four." They would continue this way as high as Spencer could count. These episodes provided a considerable amount of the overt practice Spencer had with the counting sequence—practice he could not have had without a partner.

Social routines such as these invite observational learning and imitation—processes that could start the ball rolling in number development. Such processes were evident in the vitamin excerpts, presented previously. Spencer's first comment on cardinality in this context was an exact mimicry of what had been said to him, morning after morning, for months (i.e., "Just one."). The frames he used to comment on number (e.g., "Two ____ One. Two") also started out as imitations. His babysitter had used the counting frame for weeks to label sets around the house, including sets of shoes. So, Spencer's use of this frame to label sets of shoes was no accident. It was a direct imitation of what he had heard said in the same context. He learned that you announce "Two shoes. One. Two," in the presence of shoes just like he learned that when you see a phone, you put it to your ear and start talking.

One final aspect of social scaffolding for number bears comment. Empiricist accounts of development are sometimes criticized because they seem to require effortful instruction and reinforcement on the part of the "teacher." However, potent forms of reinforcement arise in social interactions without requiring planned rewards or punishments. Many of Spencer's comments seemed to be trial balloons that he sent up to see what ideas would be accepted. An example was when he insisted that there were two frogs in the bathtub. He did not need concrete punishment, like a rat requires a shock, to know whether his thinking was on track. The correction that followed was feedback enough. Similarly, the counting game he played with his father did not originate in an explicit attempt to teach Spencer the counting sequence. It was a mutually enjoyable social activity that the two of them invented together and rewarded each other for continuing.

Blake's diary (Benson & Baroody, 2002) provides an excellent example of yet another form of positive reinforcement—the reward that comes from making

oneself understood. When he was 27 months old, his mother asked him whether he wanted to drink milk or water. He first replied, "milk," but then said, "water." Unsure how to respond, his mother asked, "Which do you want, milk or water?" Blake replied, "Two," indicating that he wanted both. His use of the number word was rewarded when he received the drinks he had requested. No further feedback or effortful instruction was required—the functionality of that word in that context was sufficient to create the behavior.

D. NUMBER DEVELOPMENT DIFFERS ACROSS INDIVIDUALS

Close range examinations of numerical development have revealed a fourth trend—different children develop the same understandings in different ways. In a longitudinal study of counting and numerical equivalence concepts, we identified two different patterns of interaction between verbal and nonverbal competence (Sandhofer & Mix, 2003). For one group of children, development was seemingly led by verbal skills (see Figure 1). These children demonstrated counting proficiency earlier than the rest of the group, with most of them accurately producing all of the small sets (2, 3, and 4) on request, at or near the beginning of the study. Even more striking was that they did so before they could match

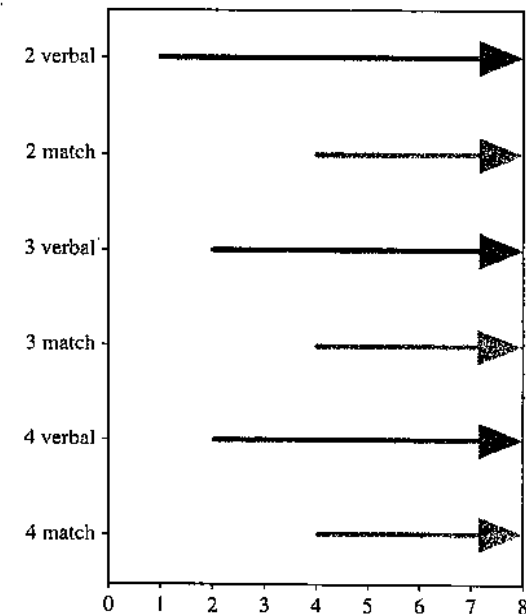


Fig. 1. Average session by which All Verbal children attained above chance performance in Sandhofer and Mix's (Sandhofer & Mix, 2003) longitudinal study of counting and numerical equivalence.

equivalent sets for *any* of the same set sizes. When they finally did match equivalent sets, an average of 2 months later, they did so for all the small set sizes at once. Thus, for these children, number development involved ascribing meaning to the small number words without forming categories for them, and then forming these categories all at once.

For a second group of children, the emergence of verbal and nonverbal skills was interleaved (see Figure 2). These children reached proficient levels for both verbal and nonverbal tasks one numerosity at a time, over a period of about 6 months. This pattern suggested that children worked out the meaning of each number word, including its corresponding equivalence class, before moving on to the next—a very different course than that obtained for children who focused on verbal skills first.

We speculated that these different patterns reflected differences in the learning histories of each child (Sandhofer & Mix, 2003). Both of our experimental tasks measured children's reasoning at a high level of abstraction. In the verbal task, children produced sets of blocks on request. In the matching task, they identified equivalent sets that were otherwise quite disparate. Verbal competence led this form of nonverbal competence in both groups, indicating that verbal skills were abstracted first and then used to abstract the children's contextualized notions of

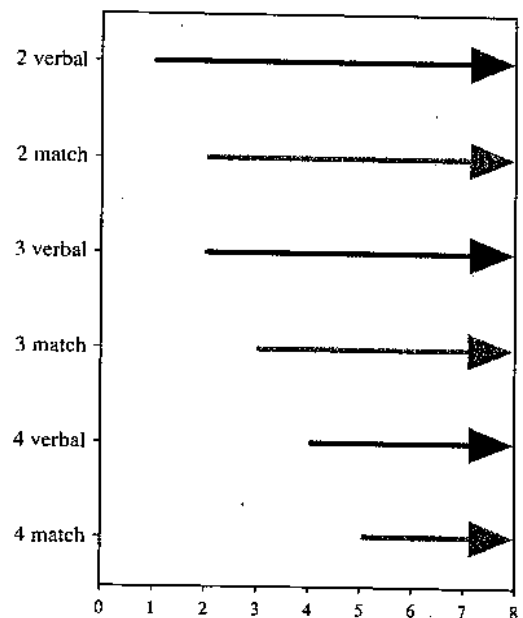


Fig. 2. Average session by which Number-by-Number children attained above chance performance in Sandhofer and Mix's (Sandhofer & Mix, 2003) longitudinal study of counting and numerical equivalence.

equivalence. The interesting difference between the two groups was that the Number-by-Number children (i.e., the second pattern) seemed to have a weak sense of number categories waiting in the wings—presumably constructed through experience with object sets and still somewhat contextualized. So, although children with this learning history were slower to abstract the verbal labels, each time they did, they were immediately able to abstract the corresponding number category as well. The All Verbal children (the first pattern) seemed to lack these weak number categories. Perhaps most of their number input had been focused on counting and number words, rather than one-to-one correspondence or play with matching sets.

E. NUMBER DEVELOPMENT IS DOMAIN GENERAL

We began this chapter by analyzing what number concepts entail, including both verbal and nonverbal components. Viewed this way, a main challenge to young learners is integrating these components—mapping one to another in a complex web of skills, situations, and ideas. Like any other mapping, numerical mappings are likely to involve noticing similarity, isolating relevant points of alignment, forming categories, and pairing words with referents. In other words, there is no reason to think that numerical development should be special, except that it may be especially difficult given the number of mappings required and the lack of obvious cues. Mix (Mix, 1999a,b, 2004a) has argued previously that domain general processes of comparison underlie the development of numerical equivalence judgments. In this section, we consider whether they also underlie verbal mappings.

The diary and longitudinal studies we have described so far provide several indications that they do. First, several trends observed in number development resemble those seen in the development of other concepts, such as color. For example, recall Wynn's (1992) finding that children realize the number words refer to numerosity before they know the specific cardinal meanings of these words. The same pattern is evident in children learning color terms. That is, they first realize that the color words as a group refer to the dimension of color (Backscheider & Shatz, 1993; Landau & Gleitman, 1985; Sandhofer & Smith, 1999). At 27 months, children asked, "what color?" respond with a color word, albeit, usually the wrong one. Within a few months, they begin to provide specific words for specific colors. For example children label red apples as "red" and blue balls as "blue," but may incorrectly label yellow balls as "purple."

Another domain general trend in number development is that local mappings precede the formation of equivalence classes. Recall that both Blake and Spencer spontaneously labeled various object sets containing the same number of items for many months before they could match equivalent sets in a forced choice task (Benson & Baroody, 2002; Mix, 2004b). The same pattern has been observed in

children learning color terms. For example, Smith (1984) reported that although 2-year-olds correctly labeled objects in terms of color, they were unable to match objects by color when the objects differed on other dimensions, such as size.

This might seem counterintuitive. After all, doesn't correctly labeling a flower, a car, and a drink with the word, "red," imply that you know that these objects are of the same color? Not necessarily. Further, research using a connectionist model demonstrated that these two senses of same are quite distinct. Smith, Gasser, and Sandhofer (1997) trained a network to label three properties of a given input. For example, given a smooth red triangle, and asked, "What color is it?" the network learned to respond "red" and when asked "What shape is it?" the network learned to respond, "triangle." However, even after learning to label objects by color the network failed to represent objects that were the same on a given property as equivalent. That is, when the network was asked, "What color is it?" and was presented with a smooth red triangle, the pattern of activation on the hidden layer was different than when the network was presented with a bumpy red square and asked "What color is it?" The network apparently failed to isolate the property of color and continued to represent aspects of the shape and texture of the objects even though these were irrelevant to the task at hand.

The order of the mappings children perform for number also reveals some interesting domain general connections. First, like number categories, color equivalence classes are affected by the degree of similarity between objects. When the target and choice objects are highly similar, for example a red airplane and a similar red airplane, even children who do not comprehend color terms can match these objects by color (Soja, 1994). When the target and choice objects are less similar or when there is competing similarity from a distractor object, children fail to match objects by color until long after they have learned to comprehend and produce color terms correctly (Rice, 1980; Sandhofer & Smith, 1999; Smith, 1984). This is precisely the same pattern we described previously for numerical equivalence judgments, number words, and object similarity.

A second ordering of interest involves first uses of the number words. Both Spencer and Blake mapped the number words onto written numerals early in development. In fact, these constituted Spencer's first number word mappings. This makes sense given that children tend to interpret new words in terms of shape as their vocabularies increase (Smith *et al.*, 2002). Indeed, children with a strong shape bias can identify more letters of the alphabet than children who lack the shape bias, presumably because learning letter names requires careful attention to shape (Longfield, 2004). When children map number words to written numerals, they may be extending the shape bias to numbers. After all, numerals have a consistent shape. Numerically equivalent sets do not.

Finally, Spencer's use of number frames is reminiscent of children's use of pivot grammar more generally. Bloom (1993) noted that children often use the same simple sentence structures to incorporate new vocabulary. For example,

they might learn the frame "Give ____" to request items and then use this frame repeatedly as they acquire new words (e.g., "Give milk," "Give toy," "Give cookie," etc.). Spencer's number frames have much the same quality. They provided a way for him to incorporate new sets into his category of "twoness."

Thus, we see many parallels between number development and development in other domains. In particular, number word learning looks quite a lot like other word learning—it starts out with loose associations between the words as a group and the broad dimension they describe; it involves overgeneralization (i.e., initially referring to many numerosities as "two"), an initial bias toward shape, and the transient use of pivot grammars; and it is built from local mappings without reference to a larger equivalence class.

The significance of these parallels is that they indicate common underlying processes. When children associate number words with the dimension of number, they are likely responding to patterns in linguistic input as they do when learning other words (Bloom & Wynn, 1997). When children map number words to shape, they are using the same strategy that works in other word learning situations (e.g., Landau, Smith, & Jones, 1988; Smith, Jones, & Landau, 1992). When they overgeneralize, they are struggling to reconcile their understanding of the underlying categories with the socially accepted categories to which words refer (e.g., Mervis, 1985). These parallels provide important insights into the mechanisms by which children integrate number language with conceptual understanding—mechanisms that we consider in Section IV of this chapter.

IV. Toward a Mechanistic Account

All current conceptualizations of numerical development hold that there is a bidirectional influence between number words and number concepts. Even those models that assume a considerable innate component concede that the mapping between verbal and nonverbal knowledge precipitates significant conceptual growth. These accounts contribute by speculating about possible fit (or lack thereof) between nonverbal and verbal representations. However, the claim itself—that verbal number maps onto nonverbal number and leads to conceptual change—is not an advance. It neither distinguishes current conceptualizations from those that came before nor provides insight into the details of how these interactions occur. In fact, these accounts may mislead by leaving the false impression that mapping number words onto number concepts is more discrete and unidirectional than it actually is.

When we look at number development close up, there are no clean, wholesale mappings from skill to understanding, from word to concept. Instead, we uncover a multitude of disconnected, local mappings, successfully achieved with a great deal of contextual and social support, gradually coalescing into a fully integrated

conceptual structure. This process can seem messy. It may be tempting to gloss over the details for the sake of theoretical clarity. However, it is from these details that we can see traces of the learning mechanisms that underlie the achievement of mature number concepts. In order to explain how number development unfolds, it will be necessary to embrace this complexity and the mechanisms it reveals.

The mechanisms revealed so far are not new and they are not specific to number. Indeed, they take us right back to the classics. For example, it is difficult to think of a better way to characterize early number development than that provided by the Vygotskian framework. In this view, learning proceeds through successive stages of socialization. Children first imitate routines that have no meaning to the child beyond their social function. Over time, children internalize these routines, and the associated language, until words, context, and concepts become inextricably merged. As children assimilate verbal procedures into their thinking, they gain access to more powerful conceptual structures that allow them to evaluate or invent new procedures. Even so, they never completely abandon nonverbal thought. Our review of early number development, particularly the diary, case, and microgenetic studies, provided many examples of this type of learning. If we want to know specifically how children make initial mappings among the verbal and nonverbal components of number, Vygotsky's ideas provide a very good start.

There is also abundant evidence of empiricist learning processes. Children often started out imitating what they had observed in specific number-relevant situations. They initially mapped words onto object sets that they could perceive—not necessarily to representations of those sets. Their attempts to generalize beyond these situations were shaped by social approval and successful communication. Individual differences in number development suggest variations in the specific patterns of input and interactions in children's learning histories. Although this may not be all there is to number development, observational learning, associative learning, and conditioning are obviously a significant part of it. Indeed, the burden facing those who would argue for domain-specific learning is to explain why empiricist processes and social scaffolding are not sufficient on their own.

Of course, simply establishing that these processes underlie development is only a start. Much remains to be learned about the specific ways they are implemented in number learning. For example, if children hook into number words by imitating social scripts, then the next step is to analyze these scripts more closely. One basic question is why children enact some scripts and not others. By isolating and comparing the situations that are numerically meaningful to children, we can determine which situational cues direct attention toward number in particular. These might include specific spatial or temporal relations, linguistic cues, or social referencing cues. Such an analysis would also indicate whether non-numerical understandings, such as recognizing similarity, or

ordering and grouping objects, help to scaffold numerical insights because different contexts will vary in the degree to which this information is either provided or required.

Another line of inquiry could focus on consistency across individuals. That is, do most children make their first number word mappings in similar contexts? Do they often name numerals, as Spencer did? Or do they first map number words to fingers, held up to represent their age, like Blake? If so, then it will be important to look especially closely at these contexts to explain their widespread use. Such universal appeal would indicate either considerable emphasis or repetition in the environment, a particularly good match to the child's cognitive capacities, or both. Perhaps there are several classes of situations different children use initially. If so, it would be possible to trace the origins of multiple pathways, similar to those we described for the case of cardinality and equivalence (Sandhofer & Mix, 2003).

Finally, researchers might ask about the structure and content of parent input. The notion that children first bring meaning to number words through imitation presupposes a major role for parents because it is they who provide social routines to imitate. If nothing else, their input sets limits on the universe of possible situations children can access. However, parent input likely makes a more profound contribution by structuring children's learning environments and directing their attention within them. For some routines, such as holding fingers up for age, parents probably teach children explicitly as part of a larger social context (i.e., occasions when new acquaintances will ask how old they are). Other routines may evolve from different social needs, such as sharing, or simply emerge as the parent comments on the environment (e.g., "Oh look! Ducks! Two ducks!"). Because these routines are pivotal in children's numerical development, it is important to know which routines parents present to children, what might lead to the creation of these routines (i.e., why these routines and not others?), and which of these routines children adopt themselves. This will tell us not only how the child's environment is structured, but also something about why it is structured that way.

It has been argued that adult intelligence is based not only on the brain but also on the environment in which the brain operates (e.g., Clark, 1997). The development of intelligence in children can be viewed the same way—as the emergence of increasingly smart, adaptive behaviors within an indivisible system of neural processes and environmental structure (Thelen & Smith, 1994). The iterative view embraces this model of conceptual change, compelling us to look beyond the boundaries of the child to explain how number concepts develop and focus instead on the close-knit interactions among numerical understanding, language, and social activity. Understanding these interactions will be time-consuming and complicated. It will require more than the typical, cross-sectional laboratory experiment, forcing us to find new and creative approaches. But the

reward will be a deeper understanding of the way number words and number concepts really interact, moment-to-moment, in all their unruly complexity.

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